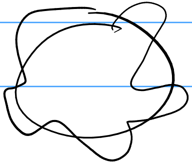
Today

- Butterfly
- IEs for PDE solving
- Theory

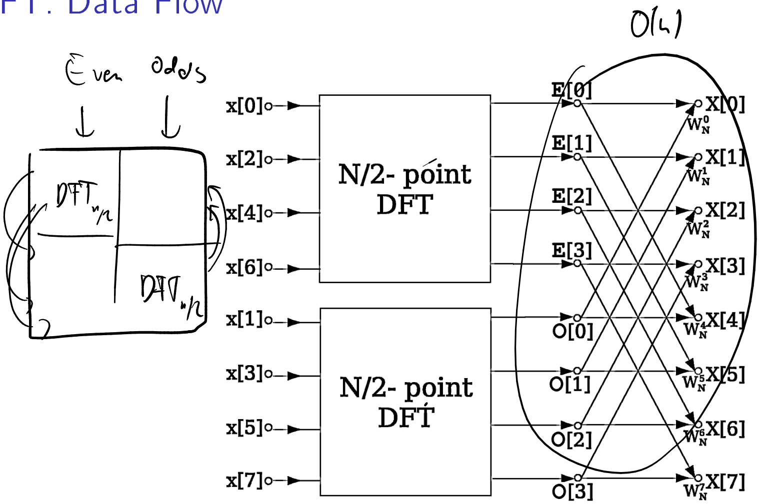
4335

Announcements

- HW3
- Projects
- Office hour moved to 4:30



FFT: Data Flow



Perhaps a little bit like a butterfly?

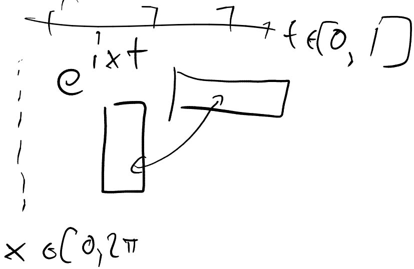
Fourier Transforms: A Different View

Claim:

> *The [numerical] rank of the normalized Fourier transform with kernel $e^{i\gamma xt}$ is bounded by a constant times γ, at any fixed precision ϵ.*

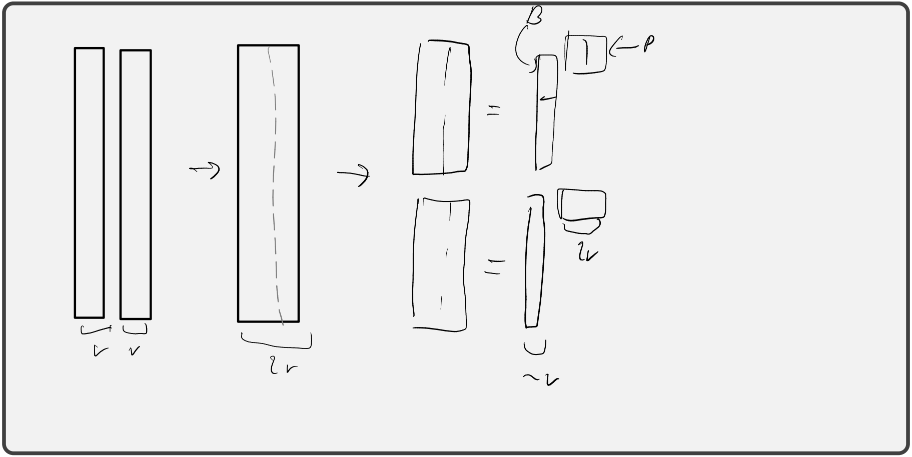
(i.e. rank is bounded by the area of the rectangle swept out by x and t)
[O'Neil et al. '10]

Demo: Butterfly Factorization (Part I)



Recompression: Making use of Area-Bounded Rank

How do rectangular submatrices get expressed so as to reveal their constant rank?



Observations

Demo: Butterfly Factorization (Part II)

For which types of matrices is the Butterfly factorization guaranteed accurate?

all of them

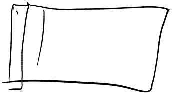
For which types of $n \times n$ matrices does the butterfly lead to a reduction in cost?

recl prop

Explore the limit cases of the characterization.

?

Observations: Cost



What is the cost (in the reduced-cost case) of the matvec?

Level 0: $P_{0,k}$: $\underbrace{r}\times\underbrace{n/2^L}$

Level ℓ: $P_{\ell,i,j}$: $r \times 2r$

Postproc. $B_{i,j}$ $n/2^L \times r$

Comments?

Outline

PDEs: Simple Ones First, More Complicated Ones Later

Laplace

$\underline{\triangle} u = 0$

$\Delta = \sum_{k=1}^{n} \partial_{x_i}^2$

- Steady-state $\partial_t u = 0$ of wave propagation, heat conduction
- Electric potential u for applied voltage
- Minimal surfaces/“soap films”
- ∇u as velocity of incompressible flow

$\partial_t u = \Delta u$

Helmholtz

$\triangle u + k^2 u = 0$

- Assume time-harmonic behavior $\tilde{u} = e^{\pm i\omega t} u(x)$ in time-domain wave equation:

$$\partial_t^2 \tilde{u} = \triangle \tilde{u}$$

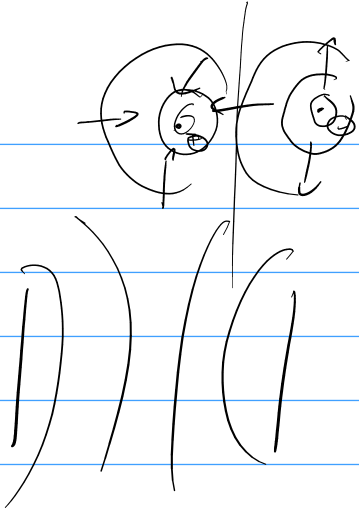
- Sign in $\tilde{u}$ determines direction of wave: Incoming/outgoing if free-space problem
- *Applications:* Propagation of sound, electromagnetic waves

$$\nabla \cdot E = \rho$$

$$E = \nabla u$$

$$\nabla \cdot \nabla u = \rho$$

$$0$$

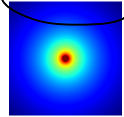


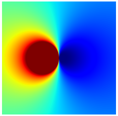
Fundamental Solutions

Laplace

$-\triangle u = \delta$

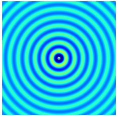
$\Delta u = f$

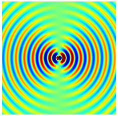


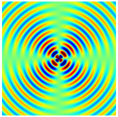


Helmholtz

$\triangle u + k^2 u = \delta$







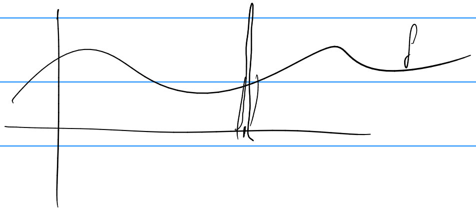
aka. *Free space Green's Functions*

How do you assign a precise meaning to the statement with the δ-function?

$$\Delta u = \delta \rightarrow \int \Delta u \, \varphi = \int \delta \varphi$$

$$f(x) \to$$

$$\delta(x) \to \int \delta(x)\, \varphi(x)\, dx$$



Green's Functions

Why care about Green's functions?

What is a non-free-space Green's function? I.e. one for a specific domain?

Green's Functions (II)

Why not just use domain Green's functions?

What if we don't know a Green's function for our PDE... at all?