September 22, 2026

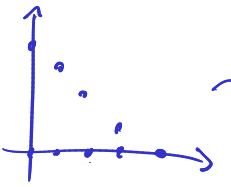
Announcements

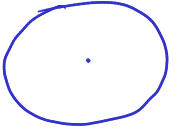
- HW2

Goals

- Expansions via linear algebra
- fast algorithms
  - → Ewald summation
  - → Barnes-Hut

Review

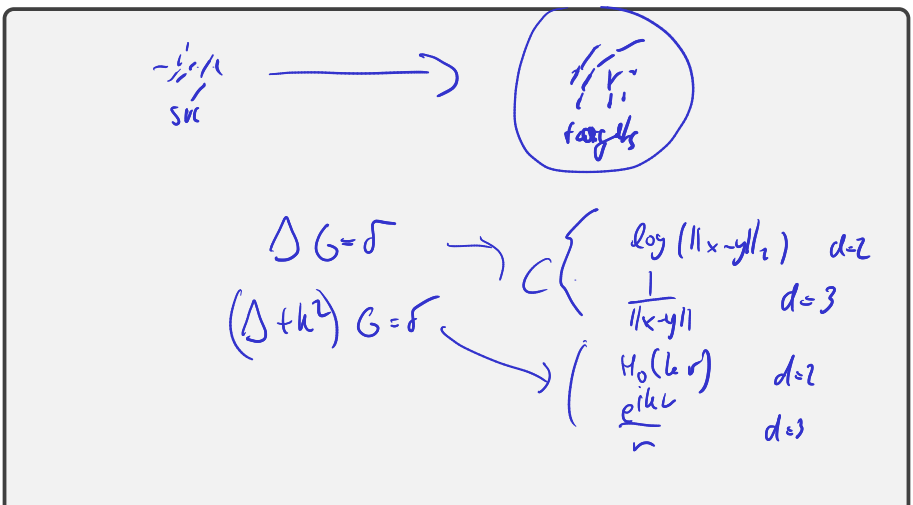




Making Multipole/Local Expansions using Linear Algebra

Actual expansions cheaper than LA approaches. Can this be fixed?

Compare costs for this situation:



The Proxy Trick

Idea: *Skeletonization using Proxies*
Demo: Skeletonization using Proxies

Q: What error do we expect from the proxy-based multipole/local 'expansions'?

Why Does the Proxy Trick Work?

In particular, how general is this? Does this work for any kernel?

Where are we now? (I)

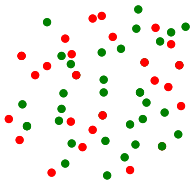
Summarize what we know about interaction ranks.

- We know that far interactions with a smooth kernel have low rank. (Because: short Taylor expansion suffices)
- If
$$\psi(\boldsymbol{x}) = \sum_j G(\boldsymbol{x}, \boldsymbol{y}_j)\varphi(\boldsymbol{y}_j)$$
satisfies a PDE (e.g. Laplace), i.e. if $G(\boldsymbol{x}, \boldsymbol{y}_j)$ satisfies a PDE, then that low rank is *even* lower.
- Can construct interior ('local') and exterior ('multipole') expansions (using Taylor or other tools).
- Can lower the number of terms using the PDE.
- Can construct LinAlg-workalikes for interior ('local') and exterior ('multipole') expansions.
- Can make those cheap using proxy points.

Where are we now? (II)

So we can compute interactions where sources are distant from targets (i.e. where the interaction is low rank) quite quickly.

Problem: In general, that's not the situation that we're in.



But: *Most* of the targets are far away from *most* of the sources.

($\Leftrightarrow$ Only a few sources are close to a chosen 'close-knit' group of targets.)

So maybe we can do business yet–we just need to split out the near interactions to get a hold of the far ones (which (a) constitute the bulk of the work and (b) can be made cheap as we saw.)