

October 1, 2026
Announcements

- HW3

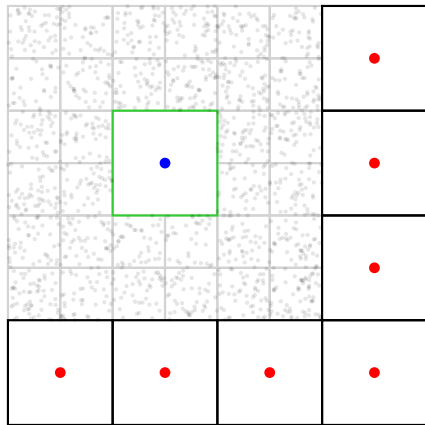
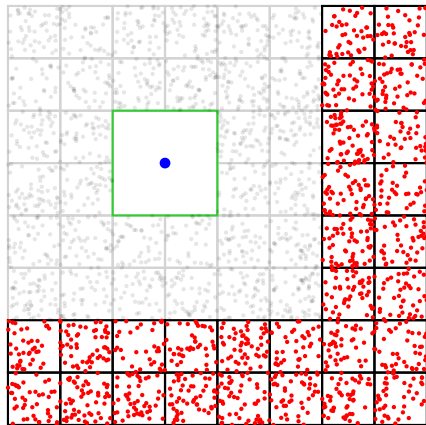
Goals

- adaptive FMM
- Interpolation
- Fourier-space method
 - NUFFT
 - Ewald
 - DMk

Review

- BH $\rightarrow$ FMM

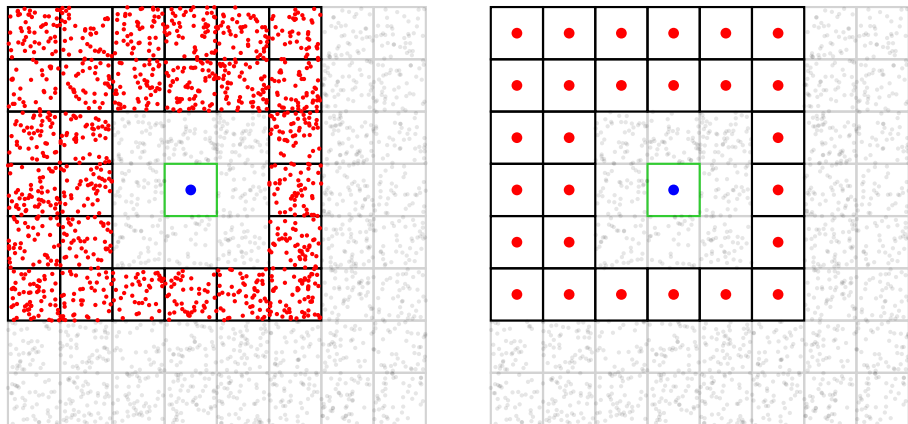
Using Multipole-to-Local



(Figure following G. Martinsson)

Come up with an algorithm that computes the interaction in the figure.

Using Multipole-to-Local: Next Level



(Figure following G. Martinsson)

Assuming we retain information from the previous level, how can we obtain a valid local expansion on the **target** box?

Define 'Interaction List'

For a box b , the interaction list I_b consists of all boxes b' so that

don't touch
same size
parents touch

Provide an upper bound on the number of elements of I_b .

$$6^d - 3^d$$

The Fast Multipole Method ('FMM')

Upward pass

1. Build tree
2. Compute interaction lists
3. Compute lowest-level multipoles from sources
4. Loop over levels $\ell = L - 1, \dots, 2$:
 - 4.1 Compute multipoles at level ℓ by $\text{mp} \rightarrow \text{mp}$

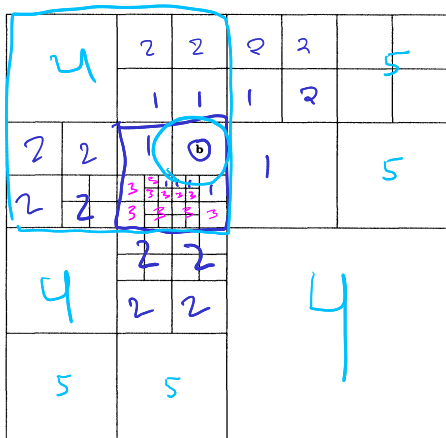
Overall algorithm: Now $O(N)$ complexity.

Note: L levels, numbered $0, \dots, L - 1$. Loop indices above *inclusive*.

Downward pass

1. Loop over levels $\ell = 2, 3, \dots, L - 1$:
 - 1.1 Loop over boxes b on level ℓ :
 - 1.1.1 Add contrib from l_b to local expansion by $\text{mp} \rightarrow \text{loc}$
 - 1.1.2 Add contrib from parent to local exp by $\text{loc} \rightarrow \text{loc}$
2. Evaluate local expansion and direct contrib from 9 neighbors.

What about adaptivity?



Adaptivity: Solution

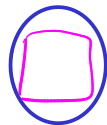
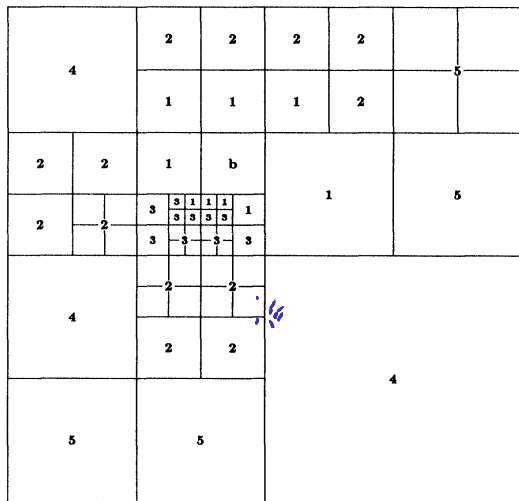
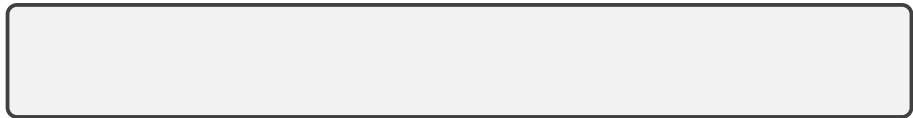


Figure credit: Carrier et al. ('88)

Adaptivity: what changes?



FMM: List of Interaction Lists

Make a list of cases:

A large, empty rectangular box with a dark border and rounded corners, intended for listing cases.

FMM: An Alternate Reality



Can we do without multipole and local expansions?

- Locals : easy
- Multipoles: scratch head,

What would be the downside here?

not obeying the PDE!

Anterpolation

WLOG: 1D. Assume K smooth, $(y_j)_{j=1}^N \subset [-1, 1]$. Let

$u(x) = \sum_{j=1}^N \rho_j K(x - y_j)$. Assume N large. How can we use interpolation to compress?

$$K(x, y)$$

$$\rightarrow K(x - y_j) \approx \sum_i U(j, i) K(x - v_i)$$

v_i : Chebyshev nodes. Let $U_{j,i}$ be interpolating from v_i to y_j .

$$\begin{aligned} u(x) &\approx \sum_j \rho_j \sum_i U(j, i) K(x - v_i) \\ &= \sum_i K(x - v_i) \sum_j \rho_j U(j, i) \\ &= \sum_i K(x - v_i) [U^T \rho]_i \end{aligned}$$

How about that smoothness assumption?

just ensure separation