September 8, 2026

Announcements

HW2

Survey last time

Goals

- redo the SVD/range finder demo
- RROR
- ID
- approx SVD ← template
- low-rank vs. smoothness

Review

What is oversampling?

$A \in \mathbb{R}^{n \times d}$

Want: rank-k approx

Natural: $Q \in \mathbb{R}^{n \times k}$

Instead: $Q \in \mathbb{R}^{n \times (k+p)}$

$QQ^T A$ could have rank $> k$

A-posteriori and Adaptivity

The HTM bound is *a-priori*. Once we're done, can we find out 'how well it turned out'?

Adaptive Range Finding: Algorithm

Outline

Rank-revealing/pivoted QR

Sometimes the SVD is too *good* (aka expensive)–we may need less accuracy/weaker promises, for a significant decrease in cost.

$A \in \mathbb{R}^{m \times n}$

$$A\Pi = QR = Q \begin{bmatrix} R_{11} & R_{12} \\ 0 & R_{22} \end{bmatrix} \begin{matrix} k \\ \\ \end{matrix}$$

Using RRQR for LRA

→ Golub / van Loan, ch. 5

- $\sigma_{k+1} \leq \|R_{22}\|_2$
- To precision $\|R\|_2$, A has at most numerical rank k.

- Complexity $O(mnk)$, requires full matrix

Interpolative Decomposition (ID): Definition

Would be helpful to know *columns of A* that contribute 'the most' to the rank.
(orthogonal transformation like in QR 'muddies the waters')

$A_{m\times n} = A_{[:,j]}\,P_{k\times n}$

j: index vector

$j = [1\ 2\ 4]^T$

- k columns of P contain only a single entry of 1.
- $|P_{ij}| \leq 2$

ID: Computation

How do we construct this (from RRQR): (short/fat case)

$$A\Pi = Q[R_{11} \; R_{12}] \qquad B = QR_{11} = A_{[:,B]} = (A\Pi)_{[:,1:k]}$$

Q: What is P, in terms of the RRQR?

$$P = [I \;\; R_{11}^{-1}R_{12}]\,\Pi^T$$

$$BP = QR_{11}[I \;\; R_{11}^{-1}R_{12}]\,\Pi^T$$

$$BP\Pi = Q[R_{11} \;\; R_{12}]\,\Pi^T$$

$$\underbrace{BP\Pi}_{A} = Q[R_{11} \;\; R_{12}]$$

ID Q vs ID A

What does row selection mean for the LRA?

$$A \approx QQ^T A$$

Now ID on Q: $Q \approx P Q_{[J,:]}$

$$A \approx QQ^T A \approx P Q_{[J,:]} Q^T A$$

$$A_{[J,:]} \approx P Q_{[J,:]} Q^T A$$

$$P A_{[J,:]} \approx P Q_{[J,:]} Q^T A$$

[Martinsson, Rokhlin, Tygert '06]

ID: Remarks

Slight tradeoff here: what?

$\approx$, but : cheap!

How would we use the ID in the context of the range finder?

just ID the sample matrix $Y = AR$

Demo: Interpolative Decomposition

Name a property that the ID has over other factorizations.