

September 8, 2026

Announcements

HLN

Buy last time

Review

Goals

- redo the SVD/range
Fluke demo
- RRQR
- ID
- approx SVD $\leftarrow$ template
- low-rank vs. smoothness

What is oversampling?

$$A \in \mathbb{R}^{n \times k}$$

Want: rank- k approx

Natural: $Q \in \mathbb{R}^{n \times k}$

Instead: $Q \in \mathbb{R}^{n \times (k+p)}$

$QQ^T A$ could have rank $> k$

A-posteriori and Adaptivity

The HTM bound is *a-priori*. Once we're done, can we find out 'how well it turned out'?



Adaptive Range Finding: Algorithm



Outline

Introduction

Dense Matrices and Computation

Tools for Low-Rank Linear Algebra

Low-Rank Approximation: Basics

Low-Rank Approximation: Error Control

Reducing Complexity

Rank and Smoothness

Near and Far: Separating out High-Rank Interactions

Outlook: Building a Fast PDE Solver

Going Infinite: Integral Operators and Functional Analysis

Singular Integrals and Potential Theory

Boundary Value Problems

Back from Infinity: Discretization

Computing Integrals: Approaches to Quadrature

Going General: More PDEs

Rank-revealing/pivoted QR

Sometimes the SVD is too *good* (aka expensive)—we may need less accuracy/weaker promises, for a significant decrease in cost.

$$A \in \mathbb{R}^{m \times n}$$
$$AT\tau = QR = Q \begin{matrix} \overbrace{\left[\begin{array}{cc} R_{11} & R_{12} \\ 0 & R_{k2} \end{array} \right]}^k \end{matrix}$$

Using RRQR for LRA

↗ Golub/van Loan, ch.5

- $\sigma_{k+1} \leq \|R_{22}\|_2$
- To precision $\|R\|_2$, A has at most numerical rank k .

• Complexity $O(mnk)$, requires full matrix

Interpolative Decomposition (ID): Definition

Would be helpful to know *columns of A* that contribute 'the most' to the rank.

(orthogonal transformation like in QR 'muddies the waters')

$$\overset{m \times n}{A} = \overset{m \times k}{A_{[:,j]}} \overset{k \times n}{P}$$

j : index vector

$$j = [1 \ 2 \ 4]^T$$

- k columns of P contain only a single entry of 1.

$$|P_{i,j}| \leq 2$$

ID: Computation

How do we construct this (from RRQR): (short/fat case)

$$A\Pi = Q[R_{11} \ R_{12}] \quad / \quad B = QR_{11} = A_{[c:,k]}^\top (A\Pi)_{[c:,k]}$$

Q: What is P , in terms of the RRQR?

$$P = [I \ R_{11}^\top R_{12}] \Pi^\top$$

$$BP = QR_{11} [I \ R_{11}^\top R_{12}] \Pi^\top$$

$$\underbrace{BP\Pi}_A = Q[R_{11} \ R_{12}] \Pi^\top$$

ID Q vs ID A



What does row selection mean for the LRA?

$$A \approx QQ^T A$$

Row ID on Q : $Q \approx PQ_{[j,:]}$

$$A \approx QQ^T A \approx PQ_{[j,:]} Q^T A$$

$$A_{[j,:]} \approx \cancel{P_{[j,:]}} Q_{[j,:]} Q^T A$$

$$PA_{[j,:]} \approx P Q_{[j,:]} Q^T A$$

[Martinsson, Rokhlin, Tygert '06]

ID: Remarks

Slight tradeoff here: what?

$\approx$, but : cheap!

How would we use the ID in the context of the range finder?

just ID the sample matrix $Y = A\alpha$

Demo: Interpolative Decomposition

Name a property that the ID has over other factorizations.