HW 2 deadline extended
HW3

Goals:
- FMN \((m2\ell, c2\ell)\)
- Solve
  - Iterative vs.
  - Direct?

Review:
- Barnes-Hut
  - \(O(N \log N)\)
  - Stages:
    - compute mpoles
      - \(V_1\): start from particle
      - \(O(N \log N)\)
    - \(V_2\):
      - only from for leaves
      - use \(m2\ell\)
      - \(O(N)\)
    - eval mpoles
    - eval direct interactions from neighbours
Using Multipole-to-Local

(Figure following G. Martinsson)

Come up with an algorithm that computes the interaction in the figure.
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- From multipoles
- Translate mode to local
- ? eval local directly
  * don’t want

Instead: reuse already computed local
Assuming we retain information from the previous level, how can we obtain a valid local expansion on the target box?
Using Multipole-to-Local: Next Level

Assuming we retain information from the previous level, how can we obtain a valid local expansion on the target box?

- Obtain multipole contributions for boxes well-separated from parent.
- Obtain multipole contributions for newly well-separated boxes (i.e., not well-separated from parent) via multipole -> local.

- Keep recursing until leaf.
- At leaf level, pick up near neighbors.
Define ‘Interaction List’

For a box $b$, the interaction list $l_b$ consists of all boxes $b'$ so that

- $b'$ and $b$ are well-separated,
- parent ($b'$) and parent ($b$) touch,
- $b'$ and $b$ are on the same level.

Size of interaction list: $\leq 27 = O(1)$
The Fast Multipole Method (‘FMM’)

**Upward pass**

1. Build tree
2. Compute interaction lists
3. Compute lowest-level multipoles from sources
4. Loop over levels \( \ell = L - 1, \ldots, 2 \):
   4.1 Compute multipoles at level \( \ell \) by \( mp \rightarrow mp \)

**Downward pass**

1. Loop over levels \( \ell = 2, 3, \ldots, L - 1 \):
   1.1 Loop over boxes \( b \) on level \( \ell \):
      1.1.1 Add contrib from \( l_b \) to local expansion by \( mp \rightarrow loc \)
      1.1.2 Add contrib from parent to local exp by \( loc \rightarrow loc \)
2. Evaluate local expansion and direct contrib from 9 neighbors.

**Overall algorithm:** Now \( O(N) \) complexity.

**Note:** \( L \) levels, numbered 0, \ldots, \( L - 1 \). Loop indices above *inclusive*.
What about adaptivity?

Figure credit: Carrier et al. (‘88)
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