Ann

Goals

- FMM for adaptive trees
- Solve ?
  - FMM + iterative
    ↳ if # iterations = $O(1)$ then linear time.
  - If it. method conv. is poor, direct solvers.
- Butterfly transform

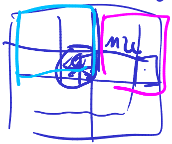
Review:

BHut: ("V2")

- Form mpole : $O(n)$
- eval mpole : $O(n \log n)$
- n = # particles

FMM:

- form / "upward pass" :
- eval / "downward pass" :
- "M2L", "L2L"



Define 'Interaction List'

target

For a box b, the interaction list I_b consists of all boxes b' so that

- b and b' are on same level
- b and b' are well-separated
- parents of b and b' touch

"we can pick up (b')'s multipole via M2L"

The Fast Multipole Method ('FMM') (non-adaptive)

Upward pass

1. Build tree ✓
2. Compute interaction lists ✓
3. Compute lowest-level multipoles from sources ✓
4. Loop over levels $\ell = L-1, \ldots, 2$:
   - 4.1 Compute multipoles at level ℓ by mp $\to$ mp ✓

Downward pass

1. Loop over levels $\ell = 2, 3, \ldots, L-1$:
   - 1.1 Loop over boxes b on level ℓ :
     - 1.1.1 Add contrib from I_b to local expansion by mp $\to$ loc
     - 1.1.2 Add contrib from parent to local exp by loc $\to$ loc
2. Evaluate local expansion and direct contrib from 9 neighbors.

Overall algorithm: Now $O(N)$ complexity.

Note: L levels, numbered $0, \ldots, L-1$. Loop indices above *inclusive.*

What about adaptivity?

b's parent

parent boxes that touch b's parent

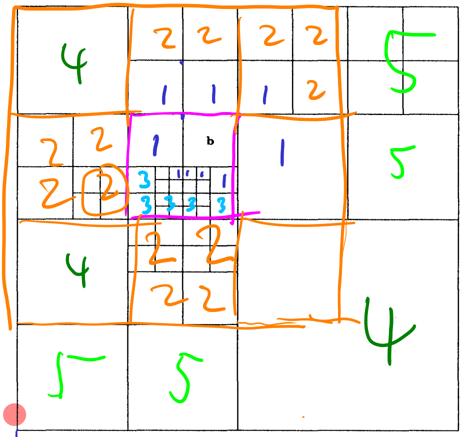


Figure credit: Carrier et al. ('88)

1 neighbor; direct

2 m2l

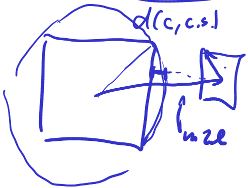
3 m2p ("eval mpole")

4 p2l

5 do nothing (l2l from parent, m2l into an ancestor)

conv. for local?

$\frac{d(c, f.t.)}{d(c, c.s.)}$



What about adaptivity?

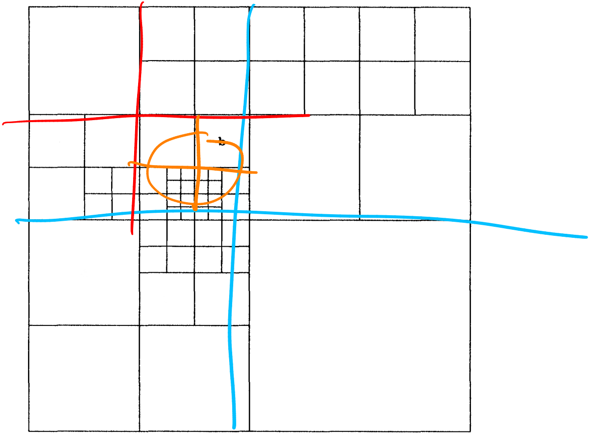


Figure credit: Carrier et al. (‘88)

What about adaptivity?

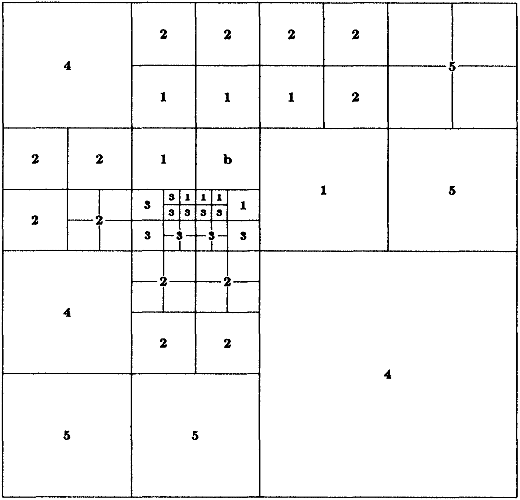


Figure credit: Carrier et al. ('88)

Adaptivity: what changes?

- MLL may miss boxes that are "too large" or "too small"

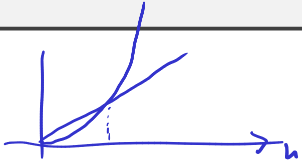
list 4

list 3

FMM: List of Interaction Lists

Make a list of cases:

1. touching/neighbor → direct
2. same-level, well-sep, parents touch → m2l
3. mpole converges, in m2l region, small → m2p
4. mpole doesn't converge, overlaps m2l region, big → p2l
5. well-sep, bigger → nothing



Outline

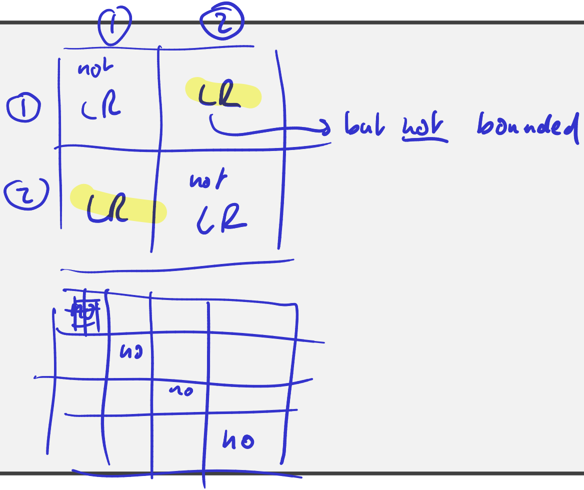
What about solving?

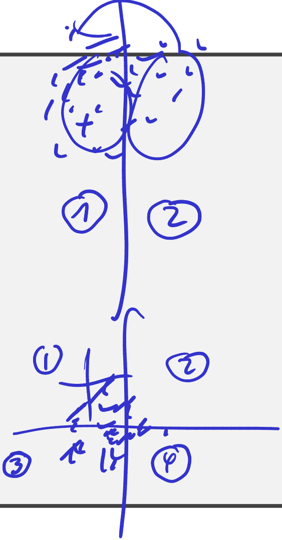
Likely computational goal: Solve a linear system $Ax = b$. How do our methods help with that?

- iterative + FMM
- direct

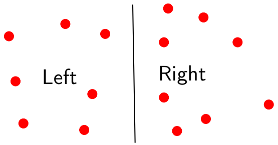
A Matrix View of Low-Rank Interaction

Only *parts of the matrix are low-rank*! What does this look like from a matrix perspective?





(Recursive) Coordinate Bisection (RCB)



Block-separable matrices

$$A = \begin{bmatrix} D_1 & A_{12} & A_{13} & A_{14} \\ A_{21} & D_2 & A_{23} & A_{24} \\ A_{31} & A_{32} & D_3 & A_{34} \\ A_{41} & A_{42} & A_{43} & D_4 \end{bmatrix}$$

where A_{ij} has low rank: How to capture rank structure?

$$A_{ij} \approx P \, \underset{\sim}{\tilde{A}_{ij}} \, T \qquad (i \neq j)$$

"upsampler"

Proxy Recap

Saw: If A comes from a kernel for which Green's formula holds, then the same skeleton will work for all of space, for a given set of sources/targets. What would the resulting matrix look like?