Announcements:

HW2

Goals:

- Do expn. stuff via NLA
- Fast alg:
  - Ewald
  - Barnes-Hut

Review:

$\int$

$\psi(x) = \sum G(x,y)\,\sigma_j$

$O(s\,k)$ $\qquad$ $O(Tk)$

$y^3 x^0 \to$

$(0,0)$ $\qquad$ $(3,0)$

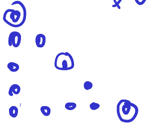
$x^3 y^0$

$\Delta u = 0$

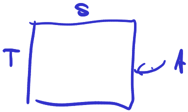
$\partial_x^2 = -\partial_y^2$

$\Delta\, \partial_x^5 \partial_y^{17} u = \partial_x^5 \partial_y^{17} \Delta u = 0$

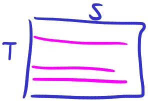
$\partial_x^7 \partial^{17} u = -\partial_x^5 \partial_y^{19} u$



~~Form the $T \times S$ matrix~~



Want: row/target subset



Idea: Use proxies to find target subset

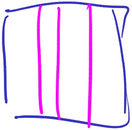
$PA_{sp} \approx A$

S

T

$$\Psi(x) = \sum G(x_i, y_j)\,\sigma_j$$

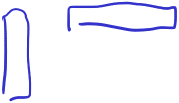
S

T  $\leftarrow A$

$$A\vec{\sigma}$$

$$\approx A_{[:,J]}\, P\, \vec{\sigma}$$

$$A \approx A_{[:,J]}\, P$$



The Proxy Trick

Idea: *Skeletonization using Proxies*

Demo: Skeletonization using Proxies

Q: What error do we expect from the proxy-based multipole/local 'expansions'?

Use # proxies terms in "good" Taylor, find accuracy

Why Does the Proxy Trick Work?

In particular, how general is this? Does this work for any kernel?

Non-PDE kernels?

\# proxies scales with $O(p^{d-1})$ not $O(p^d)$

... so a PDE is needed.

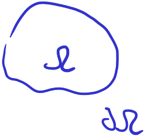
What happened?

potential represented by nearby equivalent sources

Why OK?

Green's formula

If $\Delta u = 0$ on Ω, then



$$(x\in\Omega) \qquad u(x) = \pm\int_{\partial\Omega} G(x,y)\,\frac{\partial u(y)}{\partial n}\, dS_y \pm \int_{\partial\Omega} \frac{\partial G(x,y)}{\partial n_y}\, u(y)\, dS_y$$

S_u

$D\left(\frac{\partial u}{\partial n}\right)$

Message: explicitly constructed equivalent sources

Where are we now? (I)

Summarize what we know about interaction ranks.

- We know that far interactions with a smooth kernel have low rank. (Because: short Taylor expansion suffices)
- If
$$\psi(\boldsymbol{x}) = \sum_j G(\boldsymbol{x}, \boldsymbol{y}_j)\varphi(\boldsymbol{y}_j)$$
satisfies a PDE (e.g. Laplace), i.e. if $G(\boldsymbol{x}, \boldsymbol{y}_j)$ satisfies a PDE, then that low rank is *even* lower.
- Can construct interior ('local') and exterior ('multipole') expansions (using Taylor or other tools).
- Can lower the number of terms using the PDE.
- Can construct LinAlg-workalikes for interior ('local') and exterior ('multipole') expansions.
- Can make those cheap using proxy points.