CS 598 EVS: Tensor Computations

Bayesian Methods for Tensor Decomposition

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Probabilistic Tensor Models

- Consider a tensor $\mathcal{T} \in \mathbb{Z}^{n\times n\times n}$ of count data,

$$\mathcal{T} = \sum_{l=1}^{M} \boldsymbol{e}_{i_l} \otimes \boldsymbol{e}_{j_l} \otimes \boldsymbol{e}_{k_l}$$

where (i_l, j_l, k_l) are random samples of a probability distribution $p(i,j,k)$[1]

[1] Huang, Kejun, and Nicholas D. Sidiropoulos. "Kullback-Leibler principal component for tensors is not NP-hard." 2017 51st Asilomar Conference on Signals, Systems, and Computers. IEEE, 2017.

Optimization with KL divergence

- Tensor decompositions with Kullback-Liebler (KL) divergence are suitable for approximating $p(i, j, k)$

Optimal Rank-1 Approximation with KL divergence

- The rank-1 estimate with least KL divergence is easy to compute

Probability Estimation from Samples

- Suppose we are interested in estimating a probability density $p(x, y, z)$ from samples $(x_k, y_k, z_k)_{k=1}^m$, using tensor products of orthogonal basis functions[2] $\psi_1, \ldots \psi_n$, so

$$p(x, y, z) = \sum_{i,j,k} c_{ijk} \psi_i(x) \psi_j(y) \psi_k(z)$$

[2] Tang, Xun, and Lexing Ying. "Solving high-dimensional Fokker-Planck equation with functional hierarchical tensor." Journal of Computational Physics 511 (2024): 113110.