Goals: The course objective is to give students insight and skills with computational methods as a means of investigating problems in mechanical engineering. While much of the course is based on numerical methods for differential equations, the examples and homework will be mechanics based, so that students can understand the deep connections between the behavior of mechanical systems and their numerical counterparts. Topics will include numerical solution of multidimensional boundary value problems (e.g., the heat equation, advection-diffusion, and Stokes) and initial value problems (e.g., unsteady variants of the same). By the end of the course, students will be able to identify potential sources of error and instability and be able to estimate computational costs/feasibility for these schemes. Finally, they should be able write and test (!) code to solve computational mechanics problems.

Prof: Paul Fischer (fischerp@illinois.edu)
Office Hours: Wednesday, 1-3, 4320 Siebel Center

TA: Kento Kaneko (kaneko2@illinois.edu)
Office Hours: Friday, 8-10, 28 MEB

Course Web Site: https://relate.cs.illinois.edu/course/tam470-f18/
(Homework will be posted on Relate.)


Grading: Your course grade will be based on your performance on quizzes, homework, two exams, and a final exam. Exams will be based primarily on HW and quiz questions. The course grade will be determined as follows:

- 5% Quizzes
- 50% Homework
- 25% Two exams at CBTF
- 20% Final exam

The homework assignments will include programming exercises and will be due every two weeks. Students may choose an appropriate language for the homework assignments (Matlab, Python, Fortran, C, etc.). Examples and helpful routines will be provided in Matlab. The first assignment will be posted Thursday, August 30, and due Thursday, September 13.

Homework assignments should be written legibly, with prose to explain insight drawn from the figures/exercises. Simply providing a set of plots does not complete the assignment. The plots should be on the proper scale (e.g., a lin-lin plot of $f(x) = x^{-\alpha}$ for $\alpha > 1$ on the interval $x \in [1 : 500]$ provides no insight to the value of $\alpha$; a log-log plot does.) For every problem, students should demonstrate that they have tested their code to verify that it is working correctly. Just providing a “number” without being able to justify its validity is insufficient.
Schedule – TAM 470 Fall 2018
Each topic is expected to cover anywhere from 1-3 lectures. The last topics will be subject to availability in the schedule.

1. \begin{itemize}
   \item A Computational Mechanics Example. Spring-Mass systems: stiffness matrices (Lin.Alg. I); eigenvalues (Lin.Alg. II); time advancement and time scales.
   \end{itemize}

2. \begin{itemize}
   \item 1D Interpolation: Lagrange polynomial interpolation; error and stability; differentiation; integration; BVP example.
   \end{itemize}

3. \begin{itemize}
   \item 2D Interpolation: isoparametric mappings; gradients; tensor products.
   \item Other interpolants: piecewise polynomials; splines; Fourier; Richardson extrapolation.
   \end{itemize}

4. \begin{itemize}
   \item Steady heat equation, 1D. Galerkin method: 
     .Lagrange polynomial bases .Piecewise polynomial bases (linear FEM) 
   Variable coefficients. Finite differences.
   \end{itemize}

5. \begin{itemize}
   \item Heat equation in square domains. Galerkin method: 
     Lagrange polynomial bases; piecewise polynomial bases; boundary conditions; variable coefficients. 
   Other methods: finite differences; Fourier-Galerkin.
   \end{itemize}

6. \begin{itemize}
   \item Solvers for 2D Poisson: banded solvers; fast direct solvers; iterative solvers.
   \end{itemize}

7. \begin{itemize}
   \item 2D Poisson Applications: analysis of slider bearings for read-write heads; anisotropic diffusion in 2D; solutions in deformed domains.
   \end{itemize}

8. \begin{itemize}
   \item 3D box domains. Galerkin method: Lagrange polynomial bases; piecewise polynomial bases. Finite differences.
   \end{itemize}

9. \begin{itemize}
   \item 2D Complex domains. Multi-element Lagrange polynomials ($Q_N$ bases). Matrix assembly for FEM. Iterative solvers. Schwarz methods.
   \end{itemize}

10. \begin{itemize}
    \item Unsteady heat equation: 1D: eigenmode analysis; cooling a soda can. 1D: timestepper choices; CN and L-stability. 2D: fast solvers; multidomain applications; projection in time. Multigrid.
    \end{itemize}

11. \begin{itemize}
    \item Unsteady Advection: 1D: spatial discretizations; numerical dispersion and dissipation; equivalent differential equation. 1D: timestepper choices; 2D: variable velocity fields; aliasing and stability.
    \end{itemize}

12. \begin{itemize}
    \end{itemize}

13. \begin{itemize}
    \item Unsteady Navier-Stokes: Incompressibility constraint. Rayleigh-Benard convection. Bridge-pier scouring.
    \end{itemize}
Reference Materials: