MATLAB

Applications: Statistics

CS101 Lecture #25

2016-12-05

Administrivia

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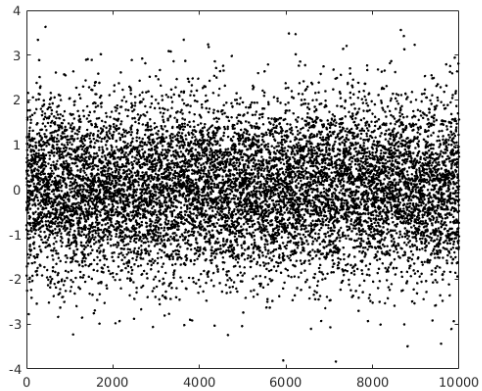
- Homework #12 is due Friday, Jan. 13.
- Final examination will be held Jan. 20, Friday 8am-11am in A-0414.

Recall: Matrix Indexing and Plot

- To refer to multiple elements of an array, use the colon operator to specify a range of the form start:end.
  - A(1:3,2) first 3 rows, 2nd column
  - A(3,:) all columns in 3rd row
- plot https://www.mathworks.com/help/matlab/ref/plot.html
- plot(Y) creates a 2-D line plot of the data in Y versus the index of each value. If Y is a matrix, then the plot function plots the columns of Y versus their row number. The x-axis scale ranges from 1 to the number of rows in Y.
- plot(X,Y) creates a 2-D line plot of the data in Y versus the corresponding values in X.

Warmup Questions

Question #1

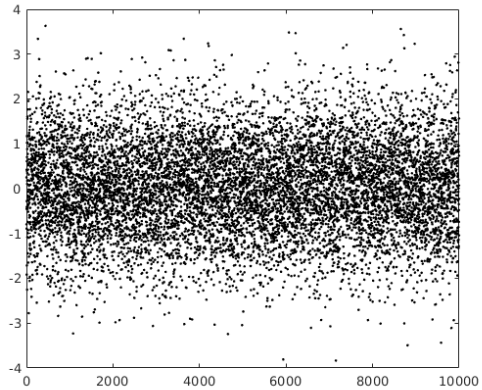


Which of the following could produce this plot?

A
```
x = rand( 10000,1 );
```
B
```
x = randi( 10000,1 );
```
C
```
x = randn( 10000,1 );
```

```
plot( x,'.' );
```

Question #1



Which of the following could produce this plot?

A `x = rand( 10000,1 );`

B `x = randi( 10000,1 );`

C `x = randn( 10000,1 );`

★

```
plot( x,'.' );
```

Question #2

$$\underline{\underline{A}}\,\underline{x} = \underline{b}$$

Which is the preferred way to solve this matrix–vector equation?

A
```
x = inv( A ) * b;
```
B
```
x = A \ b;
```
C
```
x = inv( A ) .* b;
```
D
```
x = A / b;
```

Question #2

$$\underline{\underline{A}}\underline{x} = \underline{b}$$

Which is the preferred way to solve this matrix–vector equation?

A `x = inv( A ) * b;` Why not this one? If A is a square matrix, A \ b is roughly equal to inv(A) * b, but MATLAB processes A \ b differently and more robustly.
https://www.mathworks.com/help/matlab/ref/mldivide.html
https://www.mathworks.com/help/matlab/ref/inv.html

B `x = A \ b;` ⋆

C `x = inv( A ) .* b;`

D `x = A / b;`

Question #3

```
A = [ 5 4 1- 2 2 ];
B = [ 5 4 1 -2 2 ];
```

Are `A` and `B` equal in value?

A Yes
B No

Question #3

```
A = [ 5 4 1- 2 2 ];
B = [ 5 4 1 -2 2 ];
```

Are `A` and `B` equal in value?

A Yes
B No ⋆

Example: Brexit polling

```
poll = csvread('brexit.csv');
% poll is a matrix.
% In matlab, you can use
% poll = importdata('brexit.csv');
% Then change below poll to be poll.data

plot( poll(:,2) );
plot( poll(:,3) );
% oh no! our plotted data disappeared!
```

Example: Brexit polling

```
poll = csvread('brexit.csv');
hold on;  % make plots persistent until closed
plot( poll(:,2) );
plot( poll(:,3) );
plot( poll(:,4) );
```

Example: Brexit polling

```
n = numel(poll(:,2));

mean_r = mean( poll(:,2) ) * ones( n+1,1 );
stdev_r = std( poll(:,2) );
std_rp = mean_r+stdev_r;
std_rm = mean_r-stdev_r;
hold on
plot( poll(:,2), 'ro' );
plot( 0:n,mean_r, 'r-' );
plot( 0:n,std_rp, 'r--' );
plot( 0:n,std_rm, 'r--' );
```

Example: Brexit polling

```
n = numel(poll(:,2));
mean_r = rolling_mean( poll(:,2)', 25 );
stdev_r = rolling_std( poll(:,2)', 25 );
std_rp = mean_r+stdev_r;
std_rm = mean_r-stdev_r;
hold on
plot( poll(:,2), 'ro' );
plot( 0:n-1,mean_r, 'r-' );
plot( 0:n-1,std_rp, 'r--' );
plot( 0:n-1,std_rm, 'r--' );
```

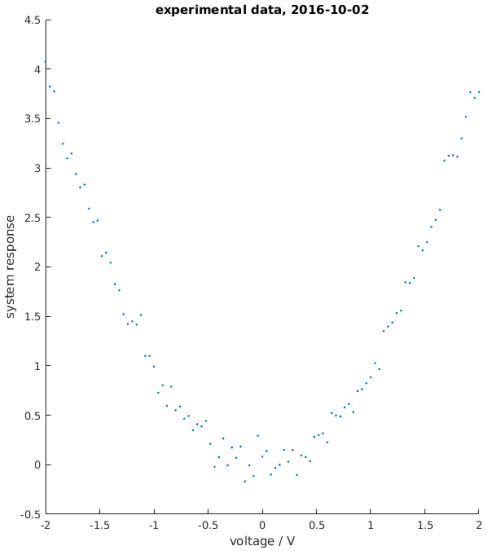
Statistics

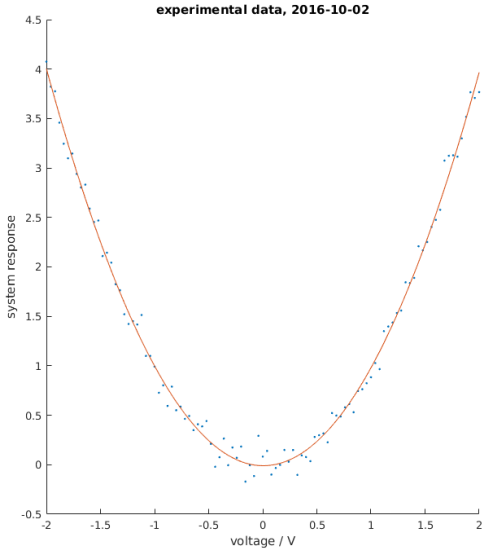
Statistical quantities

- Many operations are available:
  - `mean` (average), `median`, `std`
  - `max`, `min`, `range`
  - `iqr` (interquartile range), `corrcoef` (the correlation coefficient of two random variables is a measure of their linear dependence) (not yet supported in Octave but supported by https://octave.sourceforge.io/nan/function/corrcoef.html)
  - `sort`
  - `boxplot`, `hist`

Statistical quantities

- Often we would like to *fit* a set of data to an equation.
- We can then *interpolate* or *extrapolate*.
  - interpolate: to estimate a value within two known values in a sequence of values.
  - extrapolate: to infer something that is not explicitly stated from existing information.
- This is called *curve fitting* or *regression*.





Statistical quantities

- The simplest form of fitting is to a polynomial:

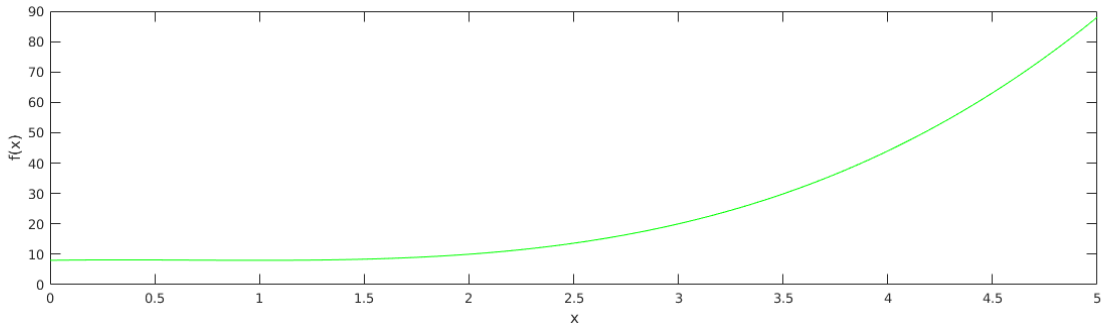
$$f(x) = a_1 x^3 + a_2 x^2 + a_3 x + a_4$$

- (Note that the numbering is a bit odd!)
- But first, we need to see how MATLAB represents polynomials.

Polynomials

$$a_1x^3 + a_2x^2 + a_3x + a_4$$

```
[ a1 a2 a3 a4 ]
```



$$x^3 - 2x^2 + x + 8$$

```
[ 1 -2 1 8 ]
```

Polynomials

$$(x^3 + x^2 + x) + (x^2 + x + 1)$$

- How would we write such an operation?

```
[ 1 1 1 0 ] + [ 0 1 1 1 ]
```

$$x^3 + 2x^2 + 2x + 1$$

Polynomials

- How can we evaluate a polynomial stored as an array?

$$f(x) = x^3 + 2x^2 + 2x + 1$$

$$f(2) = 2^3 + 2 \cdot 2^2 + 2 \cdot 2 + 1 = 8 + 8 + 4 + 1 = 21$$

```
polyval( [1 2 2 1], 2 )

polyval(p,x) returns the value of a polynomial
of degree n evaluated at x
where p is a vector of length n+1.
```

Example: Data fitting

```
x = linspace( -1,1,11 );
% linspace(x1,x2,n) generates n points
% The spacing between the points is (x2-x1)/(n-1).

y = [ 0.038 0.058 0.1 0.2 0.5 1 0.5 0.2 0.1 0.058 0.038 ];

coefs = polyfit( x,y,2 );
yfit = polyval( coefs,x );

plot( x,y,'.', x,yfit,'-' );
```

Example: Data fitting

```
x = linspace( -1,1,11 );
y = [ 0.038 0.058 0.1 0.2 0.5 1 0.5 0.2 0.1 0.058 0.038 ];

coefs = polyfit( x,y,10 );
xfit = linspace( -1.5,1.5,101 );
yfit = polyval( coefs,xfit );

plot( x,y,'.', xfit,yfit,'-' );
ylim( [-1 1] );
```

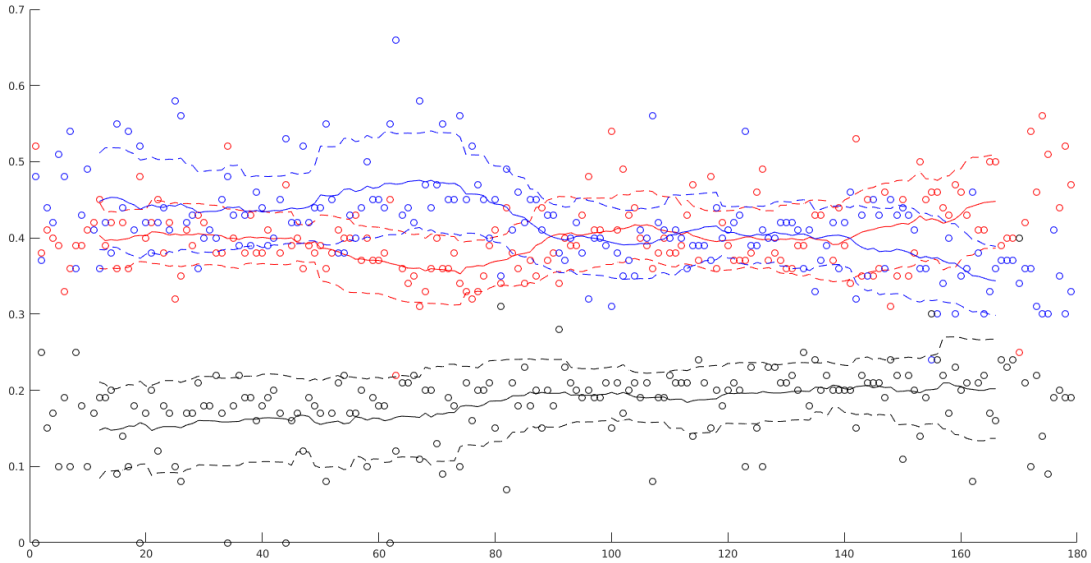
Example: Data fitting

```
x = linspace( 0,1,6 );
y = [ 1 0.5 0.2 0.1 0.058 0.038 ];

coefs = polyfit( x,y,3 );
yfit = polyval( coefs,x );

plot( x,y,'.', x,yfit,'-' );
```

Example: Brexit polling



Example: Brexit polling

```
poll = csvread('brexit.csv');
% poll is a matrix.
% In matlab, you can use
% poll = importdata('brexit.csv');
% Then change below poll to be poll.data

n = numel(poll(:,3));
mean_l = rolling_mean( poll(:,3)', 25 );

fit_poly_l = polyfit( 13:167,mean_l(13:167),19 );
poly_l = polyval( fit_poly_l,1:n );

hold on
plot( poll(:,3), 'ro' );
plot( 1:n,mean_l, 'r-' );
plot( 1:n,poly_l, 'r:' );
```

Polynomials

- Other equations are possible besides polynomials:
- See the "Nonlinear Least-Squares Curve Fitting in the Optimization Toolbox" for more information. https://www.mathworks.com/help/optim/nonlinear-least-squares-curve-fitting.html

Example: Data fitting

```
x = linspace( -2*pi,2*pi,21 );
y = sin( x );

figure; hold on;
plot( x,y,'.' );
for i = 2:9
    coefs = polyfit( x,y,i );
    xfit = linspace( -2*pi,2*pi,101 );
    yfit = polyval( coefs,xfit );
    plot( xfit,yfit,'-' );
end
```

Reminders

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